Passively Damped Structural Composite Materials Using Resistively Shunted Piezoceramic Fibers

G.A. Lesieutre, S. Yarlagadda, S. Yoshikawa, S.K. Kurtz, and Q.C. Xu

The development of damped structural materials is an area of current research with potential for high rewards. Resistively shunted piezoceramic fibers used as reinforcement in a structural composite material offer the potential to significantly increase vibration damping capability. Available data indicate the predictable nature of this electroelastic damping mechanism, an important concern in design. This article addresses the current status of an effort to develop damped composites using resistively shunted piezoceramic fibers, including modeling aspects, performance limits, design guidelines, and fabrication issues. Initial design guidelines take the form of a modified modal strain energy method. With longitudinally poled fibers, peak damping loss factors of 12% are attainable in principle, even at relatively low (30%) piezoceramic fiber volume fractions. Some 30-µm diameter piezoelectric fibers have been produced using a sol-gel method, and details of poling and shunting are under investigation.

Keywords

composite materials, passive damping, piezoelectric ceramic fibers

1. Introduction

THE use of piezoelectric materials with resistive shunting circuits to achieve passive vibration energy dissipation and resonant response reduction has been demonstrated by several researchers. [1-4] Resistively shunted piezoceramics appear to offer several advantages over more conventional approaches to passive damping, including high stiffness and damping (loss modulus) for a potential composite constituent and tailorable frequency dependence. A disadvantage includes the relatively high density of typical lead-base piezoceramics.

Because of high electroelastic coupling, the deformation of piezoelectric materials produces internal potential gradients. By placing electrodes on the material and shunting them through some finite resistance, current is allowed to flow, dissipating energy through joule heating.

When the dimensions of piezoelectric elements used for passive damping are comparable in magnitude to characteristic vibration lengths, element placement significantly affects achievable levels of structural damping. However, if the elements could be reduced in size and proliferated throughout a structure, possibly as reinforcement in a structural composite material, significant damping could be achieved with less sensitivity to placement.

This observation provided the motivation for the subject work, which addresses the development of resistively shunted piezoelectric ceramic fibers as a means to increase the vibration damping properties of structural composite materials to significant levels. Figure 1 shows a concept for such a fiber.

Key challenges identified at the outset of this effort were associated with fabrication and modeling. Fabrication issues in-

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cluded fiber production, poling, and electroding; provision of an integral, tailorable resistive path; and integration into a composite material. Modeling issues included estimation of achievable damping levels, shaping of frequency-dependent damping, and effects of complex stress states and shunting network topology. This article focuses on the modeling issues.

One of the primary goals of the modeling effort was an assessment of achievable damping performance. If initial estimates, based on simplified models, were to indicate that the levels of damping possible were not of engineering significance, there would be little motivation to proceed. If, on the other hand, further investigation was warranted, more detailed models would be needed to design composite materials for specific applications, as well as to guide continuing materials development.

The initial modeling effort concentrated on the development of simple models that could be used to establish possible performance levels. This was done using a two-step process. The first was to determine effective loss factors for the piezoelectric fiber; the second was to use those loss factors in the estimation of modal damping for flexure of a composite panel. The follow-

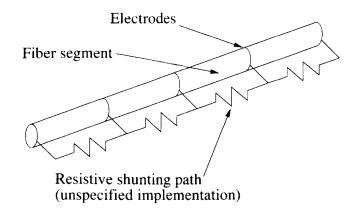


Fig. 1 Resistively shunted piezoelectric fiber concept.

ing subsections discuss each of these steps in turn, as well as limitations of the models and future efforts.

2. Damping Performance Analysis: Effective Fiber Loss Factor

To a first approximation, the peak damping of a shunted piezoelectric fiber may be estimated by assuming that individual fibers in a composite experience a simple state of stress, namely pure longitudinal tension or compression, and that the stress is approximately uniform between two electrodes on the fiber. In this situation, the maximum damping achievable in the fiber alone may be treated as a material property—an effective longitudinal loss factor.

The damping of a composite material undergoing harmonic deformation may then be estimated as the sum of the damping in the constituent materials weighted by the relative contribution of each to total strain energy. Because the fiber modulus is typically much greater than that of the matrix material, most of the strain energy of deformation (often 80 to 95%) is found in the fiber. This is one of the primary motivations for seeking ways to increase the damping of reinforcing fibers.

As discussed in Ref 4, interpretation of the operative physical dissipation mechanism as an anelastic relaxation permits the use of classical relations for analysis (such as those discussed in Ref 5) and the extension of established tools for design purposes. In this approach, the difference between the low-frequency modulus (E_p , relaxed, short circuit) and the high-frequency modulus (E_u , unrelaxed, open circuit) is closely related to the peak material damping (η , loss factor) associated with that modulus. Equations 1a and 1b express both in terms of the relaxation strength, Δ :

$$E_u = E_r(1 + \Delta) \tag{1a}$$

$$\eta = \frac{\Delta}{2(1+\Delta)^{1/2}} \tag{1b}$$

Note that the relaxation strength is closely related to the electromechanical coupling coefficient, k, as shown in Eq 2.

$$\Delta = \frac{k^2}{(1 - k^2)} \tag{2}$$

The electroelastic relaxation strength corresponding to the longitudinal modulus may be found from consideration of the material constitutive equations, specialized to the case of a single non-zero (longitudinal) stress. The constitutive equations for a linear piezoelectric material relate the stress, T, and the electric displacement, D, to the strain, S, and the electric field, E, through several material properties. These properties include the elastic moduli, c^E , the piezoelectric constants, e, and the dielectric matrix, e^S . Equation 3 shows the general form of the constitutive equations:

$$T = c^E S - e^t E$$

$$D = eS + \varepsilon^S E \tag{3}$$

Note that the equations are expressed in compressed matrix form (as opposed to tensor form). For poled piezoceramic materials, the "3" direction is taken by convention to be the direc-

$c^{E} =$	10 ¹¹ ×				-	
	1.2100	0.7540	0.7520	0	0	0
	0.7540	1.2100	0.7520	0	0	0
	0.7520	0.7520	1.1100	0	0	0
	0	0	0	0.2110	0	0
	0	0	0	0	0.2110	0
	0	0	0	0	0	0.2260
e =						
	0	0	0	12.3000	12.3000	0
	0	0	0	12.3000	12.3000	0
	-5.4000	-5.4000	15.8000	0	0	0
$\epsilon^{S} =$	10 ^{−8} ×					
	0.8107	0	0			
	0	0.8107	0			
	0	0	0.7345			

tion of poling.

The material properties for PZT-5A,^[6] of particular interest for this project, are (SI units):

The open circuit moduli, c^D , may be found by taking $D \equiv 0$:

$$c^{D} = c^{E} + e^{t}(\varepsilon^{S})^{-1}e$$
 [4]

Similarly, the short circuit modulus may be found by taking $E_3 \equiv 0$. Note that, in practice, only the component of the electric field in the direction between the electrodes is zero.

If the case of longitudinal fiber poling is considered, and the only non-zero stress is T_3 (in the direction of poling, or along the fiber axis), then the corresponding moduli are found to be

(relaxed,
$$E_3$$
 short circuit) $E_r = 5.38 \times 10^{10} \,\text{Pa}$
(unrelaxed, E_3 open circuit) $E_u = 10.41 \times 10^{10} \,\text{Pa}$

The corresponding relaxation strength and loss factor for the longitudinal modulus are then:

(relaxation strength)
$$\Delta = 0.935$$

(longitudinal loss factor) $\eta_L = 0.336$

Note that this approach yields a coupling coefficient, k_{33} , of 0.695, in agreement with the published value of 0.70.^[6]

A similar approach (E_3 shorted) may be used to address the case of radial poling and longitudinal deformation or, equivalently, longitudinal poling and transverse deformation, to yield a value for the transverse loss factor:

(transverse loss factor)
$$\eta_T = 0.081$$

Note that these values for η_L and η_T are ideal, peak loss factors attainable in practice over a limited frequency range. In composite design, other factors affecting the frequency-dependent

behavior must also be addressed. These factors include resistive shunting, nonuniform strain over finite fiber segment lengths, and shunting network topology. The following subsections briefly address the general effects of each of these factors.

2.1 Resistive Shunting

The dynamics of the electrical RC shunting network result in frequency-dependent behavior. However, the value of the shunt resistance(s) may be specified by a designer to tailor or "tune" the frequency dependence of damping to the application. Using the complex modulus representation of material properties (E = E' + jE''), and assuming j multiple discrete electroelastic relaxations, the frequency dependence of the piezoceramic storage modulus, E', the loss modulus, E'', and the loss factor, η , are given by: [4]

$$E'(\omega) = E_r \left[1 + \sum_{i \text{ relaxations}} \Delta_i \frac{(\omega \tau_{\varepsilon i})^2}{1 + (\omega \tau_{\varepsilon i})^2} \right]$$
 [5a]

$$E''(\omega) = E_r \sum_{i \text{ relaxations}} \Delta_i \frac{(\omega \tau_{\varepsilon i})}{1 + (\omega \tau_{\varepsilon i})^2}$$
 [5b]

$$\eta(\omega) = \frac{E''(\omega)}{E'(\omega)} = \eta_{\text{max}} \frac{2(\omega \overline{\tau})}{1 + (\omega \overline{\tau})^2}$$
(for a single relaxation) [5c]

where $\tau_{\epsilon i}$ is the *i*th characteristic relaxation time at constant strain, and $\bar{\tau}$ is the relaxation time (for a single relaxation):

$$\overline{\tau} = \tau_{\varepsilon} (1 + \Delta)^{1/2}$$
 [5d]

and $\tau_{\epsilon i}$ is given by, for a single piezo segment:

$$\tau_{ei} = R_i C_i^{S} \tag{5e}$$

where R_i is the shunting resistance, and C_i^S is the capacitance at constant strain (between the two electrodes on the single segment).

2.2 Nonuniform Strain in Fiber Segments

If the strain within a fiber segment between adjacent electrodes changes sign within the segment, the effective loss factor approach to estimating damping is inappropriate. For example, consider the case of a sinusoidal strain distribution. If the wavelength is on the order of the segment length, no voltage appears across the electrodes, and as a result, no damping can occur. However, the effective loss factor approach would predict some damping based on the non-zero strain energy stored in the segment. In practice then, the fiber segment lengths must be considerably shorter than the smallest wavelength of vibration to be damped. Note that this effect is a result of the fact that, with external resistive shunting, piezoelectric damping is not

an intrinsic property of the material, but an extrinsic one, depending on structural length scales.

2.3 Shunting Network Topology

As noted in the preceding, the dynamics of the electrical shunting circuit results in frequency-dependent behavior. A circuit comprising a piezoelectric fiber (electrically a capacitor) and a shunt resistor exhibits characteristic exponential relaxation. The time constant of this RC circuit relaxation can be tuned to produce peak damping at a frequency of interest through the suitable selection of the shunt resistance.

For single-segment (monolithic) piezoelectric elements, this selection is fairly straightforward. For more complex circuits with multiple segments experiencing nonuniform strain, deformation of adjacent segments affects the electrical impedance "observed" at the terminals of a given segment. This factor should also be considered in design. Again, ensuring that fiber segment lengths are considerably shorter than the smallest wavelength of interest should minimize this effect.

3. Performance Analysis: Fiber Effectiveness in Composite Damping

The objectives of this part of the effort were (1) to develop a theoretical model for prediction of the modal damping of polymer/matrix composite plates with added resistively shunted piezoelectric fibers and (2) to use this model to assess the potential effectiveness of such fibers in damping plate vibrations. Previous study of the flexure of composite panels has shown that, in general, the reinforcing fiber (whatever the material) plays a significant role in damping "fiber-dominated" bending modes, but is less effective in damping "matrix-dominated" twisting modes. [7]

A two-part approach was followed in modeling composite plate damping. The first part involved micromechanical modeling to predict the stiffness and damping properties of a single composite lamina from fiber and matrix properties. The second part involved analytical dynamic modal analysis of a midplane-symmetric laminated composite plate.

3.1 Lamina Modeling

Several micromechanical models for calculating lamina elastic properties from constituent properties for two-phase composites are available in the literature. However, in this work the composite comprises three phases: reinforcing glass fibers, epoxy matrix, and resistively shunted piezoelectric fibers

The three-phase composite of interest was treated as a twophase material by considering the reinforcing fibers and matrix as an effective matrix phase and the piezoelectric fiber as the reinforcing phase. Properties of the effective matrix material were then calculated using micromechanical models valid for isotropic fibers in an isotropic matrix. Because of its two-phase nature, the resulting effective matrix is transversely isotropic.

The piezoelectric fibers also were treated as transversely isotropic, requiring the use of a micromechanical model that is valid for transversely isotropic fibers and matrix to determine

Table 1 Constituent material properties

	3M S2-glass fiber	Hercules 3501-6 epoxy
Young's modulus, E, GPa	86	4.0
Shear modulus, G, GPa	35	1.45
Poisson's ratio, v	0.22	0.38
Young's loss factor, η_F	0	0.03
Shear loss factor, η_G	0	0.033

Table 2 Composite plate configuration

LayupGeometry	

lamina properties. One suggested scheme^[8] was used to transform a micromechanical model for transversely isotropic fibers and an isotropic matrix^[9] for use with transversely isotropic fibers and matrix.

The lamina elastic models described in the preceding were extended to include damping by representing material damping properties in terms of complex moduli. For harmonic forced vibration of a viscoelastic material, effective dynamic moduli can be determined from expressions for elastic moduli by allowing the elastic moduli to be complex. [10] The real part of a complex modulus is a measure of the stiffness of a material, whereas the complex part is a measure of damping.

3.1.1 Material Properties

Table 1 summarizes the properties used for the constituent materials in subsequent analysis. [11-13] Piezoelectric fiber damping was characterized by the (maximum) effective loss factors described in the preceding section. Note that the glass fiber has been assumed to be lossless, whereas the epoxy matrix exhibits substantial damping.

3.2 Plate Modal Analysis

The modal frequencies and damping of the composite plate were determined from lamina stiffness and damping properties, laminate stacking sequence and ply orientation, and plate geometry and boundary conditions. The approach used in this work is similar to that described in Ref 7, which contains additional references to other pertinent work. The following paragraphs provide an overview of the approach.

A higher order plate theory was used to include shear deformation and rotary inertia effects; appropriate terms appear in the expressions for plate strain and kinetic energy.

The Rayleigh-Ritz method was used to model the dynamic behavior of the plate under various boundary conditions. The assumed plate mode shapes for both transverse displacements and shear deformation were combinations of simple beam mode shapes that were appropriate to the boundary conditions. Use of these shapes with the strain and kinetic energy expressions and subsequent minimization yielded a complex matrix eigenvalue problem. The eigenvalues and eigenvectors were calculated using a standard complex eigenvalue extraction routine.

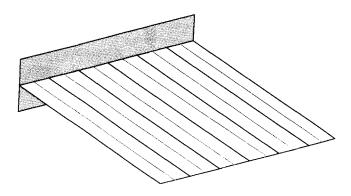


Fig. 2 Composite plate configuration.

The resulting complex eigenvalues had the form:

$$\lambda = -\zeta \omega \pm i \omega \sqrt{1 - \zeta^2}$$
 [6]

from which the damped modal vibration frequency, $\omega\sqrt{1-\zeta^2}$, and modal damping ratio, ζ , were readily determined. Note that in the simplest case of a structure made from a single lightly damped material, the modal damping ratio is approximately half the material loss factor, η , at the corresponding frequency. An effective modal loss factor may thus be defined as twice the modal damping ratio.

3.2.1 Plate Configuration

Table 2 and Fig. 2 illustrate the baseline layup, geometry, and boundary conditions considered in this analysis.

3.3 Results

The dependence of plate modal damping on piezoelectric fiber volume fraction was of particular interest in this study. Initially, the only piezoelectric damping of interest was that due to longitudinal stress, with a corresponding loss factor of 0.34 (longitudinal poling). Although it might seem straightforward to evaluate the damping of such composite materials assuming that the only non-zero damping loss factor is associated with longitudinal stress, such an approach would be incorrect. One result of such an assumption is a negative loss factor associated with some material deformation, which is not physically possible. This analysis used a non-zero loss factor associated with transverse deformation and assumed that it acted in addition to the longitudinal loss factor. For longitudinal fiber segment poling, the minimum value required was 0.05, whereas that determined from the material constitutive equations (with E_3 shorted) was 0.08.

This approach to combining damping associated with longitudinal and transverse deformation yields an upper bound for the damping. Note that in practice, because each fiber segment has only a single pair of electrodes, transverse deformation may increase or decrease the potential difference between them, depending on the sign of the transverse stress. This would change the apparent damping, increasing or decreasing it according to the relative signs of the stresses. In general, the inclusion of transverse damping changed the results by only 1

Table 3 Analysis cases for longitudinal fiber damping

		Volume fraction		
Case No.	Piezo fiber	Glass fiber	Epoxy matrix	
0	0	0.6	0.4	
1	0.1	0.5	0.4	
2	0.2	0.4	0.4	
3	0.3	0.3	0.4	

Table 4 Elastic properties of 0 and 30% piezo lamina

	Case 0	Case 3
Longitudinal modulus, E ₁ , GPa	53.2	43.4
Transverse modulus, E_T , GPa	13.5	13.7
Shear modulus, G_{LT} , GPa	5.0	5.2
Transverse shear modulus, G_{TT}	4.5	4.6
	0.28	0.33
Poisson's ratio, v_{LT} Density, ρ , kg/m ³	2006	3581

Table 5 Modal loss factors for 0% and 30% piezo lamina

Mode	Case 0	Case 3	Increase, %	
1	0.0018	0.1230	6730%	
2	0.0167	0.0684	310%	
3	0.0240	0.0354	48%	
4	0.0021	0.1210	5660%	
5	0.0078	0.0977	1150%	
6	0.0250	0.0624	150%	

to 3%. A more detailed analysis of the effects of complex stress states would likely require a numerical coupled field approach, perhaps like that described in Ref 14.

In all of the cases considered, the epoxy matrix volume fraction was assumed to be fixed at 0.40, whereas the remaining 0.60 was divided between glass fiber and resistively shunted piezoelectric fiber. A reasonable upper limit on piezoceramic volume fraction was taken as 0.30. Table 3 summarizes the initial analysis cases considered.

Table 4 summarizes the calculated elastic properties for cases 0 and 3 (0 and 30% piezo, respectively). Note that the addition of 30% piezoceramic fiber decreases the plate longitudinal (short circuit) stiffness by 18% and increases the density by 79%. Clearly, large increases in damping are needed to justify the use of piezoceramic fibers in a glass/epoxy composite structure.

Figures 3 and 4 show the dependence of composite plate vibration frequencies and damping on piezoelectric fiber volume fraction. Note that the modal frequencies generally decrease with the addition of piezoceramic fiber and that the modal loss factors increase dramatically.

The frequencies decrease because the density increases and the stiffness decreases. With the addition of 30% piezoceramic fiber, the largest frequency change is 31%, whereas the smallest is 25%. By sensitivity, the modes are ordered 1, 4, 5, 2, 6, and 3. Mode 1 is a fiber-dominated bending mode, whereas mode 3 is a matrix-dominated twisting mode. The behavior of the other modes is between these extremes.

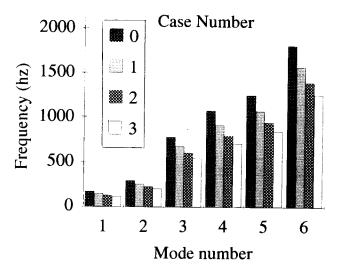


Fig. 3 Modal vibration frequencies versus mode number and piezo volume fraction case.

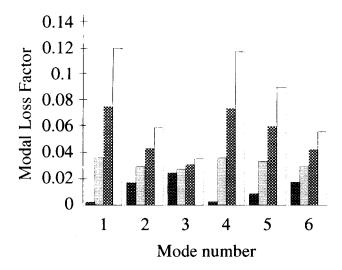


Fig. 4 Modal loss factors versus mode number and piezo volume fraction case.

Table 5 summarizes the effective modal loss factors for cases 0 and 3 (0 and 30% piezo, respectively), along with the percentage changes in damping. With the addition of 30% piezoceramic fiber, mode 1 damping increases by a factor of 70 to a level of over 12%. Mode 3 is affected the least, but nevertheless increases by 50% to a level of 3.5%. This result is consistent with previous findings regarding the role of the fiber in composite damping.^[7] Note that, for nonunidirectional fiber arrangements and for shell-type structures that carry in-plane loads, fiber damping would play an important role in most vibration modes.

Additional cases were also considered, including both longitudinal and radial poling of piezo-coated glass fibers. In all cases, the dominant contributor to increased composite damping was the loss factor associated with longitudinal fiber deformation. This is primarily the result of the high participation in composite modal strain energy.

3.4 Experimental Results

Experimental results validating the general concept of shunted piezoelectric damping using monolithic elements (including small, 1-mm diam, PZT tubes) have been reported in the literature. [1-4] No fine-scale resistively shunted piezoelectric fiber composites have been constructed to date, although work on such materials is in progress.

4. Piezoelectric Fiber Fabrication

Researchers have recently fabricated 30-µm diameter PZT fibers (PbZr_{0.52}Ti_{0.48}O₃) from viscous spinnable solutions prepared by sol-gel processing of alkoxide precursors. Following heat treatment, these fibers were found to consist of fully dense submicron grains and to exhibit dielectric constants of 800. This progress is discussed in detail in Ref 15.

The additional critical aspects of fiber electroding, poling, and integral shunting are currently under investigation. Various alternative approaches are being considered for each.

5. Summary and Conclusions

In summary, there is a need for structural materials with enhanced intrinsic vibration damping capability. Although researchers have recently demonstrated the use of resistively shunted piezoelectric materials to increase structural damping, these efforts used elements with dimensions on the same order as those of structural elements. The extension of this emerging research area to composite materials with tailorable frequency-dependent damping, along with corresponding design and analysis tools, shows promise. With longitudinally poled fibers, peak modal loss factors of 12% are theoretically attainable in a PZT/S-glass/epoxy composite, even at relatively low (30%) piezoceramic fiber volume fractions.

To the extent practical, piezoelectric fiber material should be poled longitudinally to attain the maximum damping; the form of the material (coating versus separate fiber) is unimportant. Successful pursuit of this avenue of development would mark a significant advance in the technology of engineered structural materials.

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